# M.Sc. 4th Semester Examination, 2022
# Department of Mathematics,
# Mugberia Gangadhar Mahavidyalaya
# (Functional Analysis)
# Paper MTM – 401

**FULL MARKS: 50 : : Time : 02 hours**

(***Candidates are required to give their answers in their own words as far as practicable***)

**1. Answer any four questions** **2×4=8**

(a) Prove that the norm function is continuous. **2**

(b) Give an example of a normal operator which is not self-adjoint. **2**

(c) Let X be a normed space. Show that $x_n \to x$ weakly in X does not imply $x_n \to x$ in X in general. **2**

(d) Show that every normed space can be embedded as a dense subspace of a Banach space. **2**

(e) Is the space $l^p$ with $p \neq 2$ is an Inner product space? Justify. **2**

(f) Give an example of incomplete metric space. **2**

**2. Answer any four questions** **4×4=16**

(a) If $\{e_1, e_2, e_3, \ldots, e_n\}$ is a finite orthonormal set in an inner product space X and $x$ is any element of X, then prove that$\sum_{j=1}^{n} |\langle x,\ e_j \rangle|^2 \leq ||x||^2$. When equality holds? **4**

(b) Show that $< Ae_j, e_i >= (i + j + 1)^{-1}$ for $0 \leq i, j \leq \infty$ defines a bounded operator on $l^2(\mathbb{N} \cup \{0\})$ with $\| A \| \leq \pi$. **4**

(c) Show that the space $l^p$is complete; here $p$ is fixed and $1 \leq p < +\infty$. **4**

(d) Let $S \in BL(H)$, where $H$ is a Hilbert space. Prove that for all $x, y \in H$,

$$< Sx, y >= \frac{1}{4} \sum_{n=0}^{3} i^n < S(x + i^n y), (x + i^n y) >.$$ **4**

(e) What is Canonical Embedding? Show that it is linear. **4**

(f) Give an example to show that the completeness of the domain is an essential requirement in the Uniform Boundedness Principle. **4**

**3. Answer any two questions** **8×2=16**

(a) (i) If $X$ is a normed space, $M$ is a closed subspace of $X$, $x_0 \in X\setminus M$ and $d = dist(x_0, M)$, show that there is an $f \in X^*$ such that $f(x_0) = 1, f(x) = 0$ for all $x \in M$ and $\|f\| = d^{-1}$. **5+3**

(ii) Let S be the Unilateral shift operator. Show that ${S^*}^n \xrightarrow{S} 0$ but not uniformly.

(b) (i) Let $P \in BL(\mathcal{H})$ be a nonzero projection on a Hilbert space $\mathcal{H}$ and $\|P\| = 1$. Then show that P is an orthogonal projection. **4+4**

(ii) Show that $Ran(T) = Ran(T^*)$ if $T \in BL(H)$ is normal and $H$ is a Hilbert space.

(c) (i) Prove that in a finite dimensional normed space every linear operator is bounded. **3+5**

(ii) Show that in a normed space a linear operator is continuous if and only if it is bounded.

(d) Prove that an absolutely convergent series is convergent iff the space is Banach space. **8**

**[Internal Assesment-10 marks]**

# M.Sc. 4th Semester Examination, 2022
## Department of Mathematics,
## Mugberia Gangadhar Mahavidyalaya
## (Fuzzy Mathematics with Applications and Soft Computing)
## Paper MTM – 402

**FULL MARKS: 50 : : Time : 02 hours**

(***Candidates are required to give their answers in their own words as far as practicable***)

Unit I: **Fuzzy Mathematics with Applications**

**1. Answer any two questions** **2×2=4**

(a) Define a fuzzy multi-objective linear programming problem in general form. **2**

(b) Define trapezoidal and Gaussian fuzzy number. **2**

(c) Evaluate the following: $2(5,6,8,12)+3(-1,3,4)-5(-3,2)+8$. **2**

(d) What do you meant by symmetric and non-symmetric fuzzy LPP? **2**

**2. Answer any two questions** **4×4=8**

(a) Illustrate Zadeh's Extension principle. Use it, show that 7-4=3. **4**

(b) Show that fuzzy sets do not satisfy laws of contradiction and excluded middle. **4**

(c) If $\tilde{A}\tilde{Y}=\tilde{B}$be a fuzzy equation, find the solution $\tilde{Y}$ such that the membership of $\tilde{A}$ and $\tilde{B}$ are as follows: **4**

$$\mu_{\tilde{A}}(x)=\begin{cases}0, & x\leq 3 \; and \; x>5\\ x-3, & 3<x\leq 4\\ 5-x, & 4<x\leq 5\end{cases}$$

$$\mu_{\tilde{B}}(x)=\begin{cases}0, & x\leq 12 \; and \; x>32\\ (x-12)/8, & 12<x\leq 20\\ (32-x)/12, & 20<x\leq 32.\end{cases}$$

(d) Graphically explain how a triangular fuzzy number $\tilde{A} = (1, 5, 13)$ can be expressed in the form $\tilde{A} = \cup \{\alpha A_\alpha : 0 < \alpha \leq 1\}$, where $\cup$ denotes the standard fuzzy union, $\alpha A_\alpha$ is a special fuzzy set define as $\mu_{\alpha A_\alpha}(x) = \alpha \wedge \chi_{A_\alpha}(x)$ and $\chi$ is a characteristic function of a crisp set. **4**

**3. Answer any one question** **8×1=8**

(a) (i) If $\tilde{A}$ = "real number considerably larger than 10" where, **2+6**

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq 10 \\ (1 + (x - 10)^{-2})^{-1}, & x > 10 \end{cases}$$

Find $A_\alpha$ (α-level set) when $\alpha = 0.50$

(ii) Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers whose membership are given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x+2)/2, & -2 < x \leq 0 \\ (2-x)/2, & 0 < x < 2 \\ 0, & otherwise \end{cases}$$

$$\mu_{\tilde{B}}(x) = \begin{cases} (x-2)/2, & 2 < x \leq 4 \\ (6-x)/2, & 4 < x \leq 6 \\ 0, & otherwise. \end{cases}$$

Calculate the fuzzy number $\tilde{A} + \tilde{B}$.

(b) (i) Illustrate the Bellman and Zadeh's principle for fuzzy LPP. **2+6**

(ii) Explain Zimmermann's method to convert the fuzzy LPP to crisp LPP.

**[Internal Assesment-05 marks]**

## Unit II: **Soft Computing**

**1. Answer any two questions** **2×2=4**

(a) Write the features of soft computing. **2**

(b) Find the max-min composition of the following fuzzy relations **2**

$$\tilde{R}_1: \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.7 \\ 1 & 0 \end{bmatrix} \end{array} \text{ and } \tilde{R}_2: \begin{array}{c} \\ y_1 \\ y_1 \end{array} \begin{array}{c} \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0.8 & 0.6 \\ 0.5 & 0 & 0.2 & 0.5 \end{bmatrix} \end{array}$$

(c) What are the disadvantages of binary coded Genetic Algorithm? **2**

(d) What do you mean by supervised and unsupervised learning? **2**

**2. Answer any two questions** **4×2=8**

(a) Let $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c\}$ be two universes of discourses. Also, let $\tilde{A} = \{(1, 0.2), (2,0.5), (3,0.7), (4,1.0)\}$, $\tilde{B}$={(1,0.3),(2,0.4),(3,0.8),(4,0.7)} and $\tilde{C} = \{(a, 0.1), (b, 0.6), (c, 0.9)\}$. Determine the fuzzy relation of the following fuzzy rule "IF x is $\tilde{A}$ AND x is $\tilde{B}$ THEN y is $\tilde{C}$ ". **4**

(b) Find the relational matrix of the concept "a young tall man", where "Young man"= $\frac{0}{115} + \frac{0.5}{120} + \frac{1}{125} + \frac{0.5}{130} + \frac{0}{135}$ and "Tall man"= $\frac{0}{170} + \frac{0.5}{175} + \frac{1}{180} + \frac{1}{185} + \frac{1}{190}$, if possible with reason. **4**

(c) Generate the output of logical OR function by McCulloch-Pitts **4** neuron model.

(d) Explain linearly separable and linearly non-separable patterns? Find the weights and threshold values that should classify the following input/ output pairs **4**

| $x_1$ | $x_2$ | $y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**3. Answer any one question** **8×1=8**

(a) (i) Draw a flow chart of Genetic Algorithm. **3+5**

(ii) Select the parent chromosomes for crossover using Roulette wheel selection procedure for the following information. Objective function: Max $f(x) = 50x - x^2, 1 \leq x \leq 30$, Current population: 01011, 10011, 01110, 01010, 01101 Random numbers: 0.41, 0.97, 0.12, 0.36, 0.64

(b) Write the procedure of perceptron neural network for single output class. Use it to adjust the weights and bias of logical OR function with initial weights $W = [0, 1]^T$ and bias $b = 0$. **3+5**

**[Internal Assesment-05 marks]**

# 0 M.Sc. 4th Semester Examination, 2022
# Department of Mathematics,
# Mugberia Gangadhar Mahavidyalaya
# (Magneto Hydro-Dynamics and Stochastic Process and Regression)
# Paper MTM – 403

**FULL MARKS: 50 : : Time : 02 hours**

(***Candidates are required to give their answers in their own words as far as practicable***)

## Unit I: **Magneto Hydro-Dynamics**

**1. Answer any two questions** **2×2=4**

(a) Define the term magnetic diffusivity. **2**

(b) Write down the working procedure of 'magneto-fluid-dynamics (MFD) submarines'. **2**

(c) Define Reynolds number and explain its significance. **2**

(d) Write Navier stokes equation of motion. **2**

**2. Answer any two questions** **4×2=8**

(a) Write down the basic equations of magneto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows. **4**

(b) Prove that in a steady non-uniformly rotating star, the angular velocity must be constant over the surface traced out by the rotation of the magnetic lines of force about the magnetic field axis. **4**

(c) Prove that for a conducting liquid, the flux of the magnetic field through a closed circuit of the fluid particles moving along with the fluid is constant for all time. **4**

(d) Define the terms Alfven's velocity and Alfven's waves. Hence, derive the speed of propagation is $\sqrt{c^2 + V_A^2}$ for magneto hydrodynamic wave, where symbols have their usual meaning. **4**

**3. Answer any one question** **8×1=8**

(a) (i) Show that magnetic body force per unit volume for a conducting fluid in a magnetic field is equivalent to a tension per unit area along the lines of force, together with a hydro-static pressure. **6+2**

(ii) What is finch effect?

(b) A viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate $z = - L$ (lower) and a horizontal infinitely long non-conducting plate $z = L$ (upper). Assume that a uniform magnetic field $H_0$ acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the magnetic field. **8**

**[Internal Assesment-05 marks]**

Unit II: **Stochastic Process and Regression**

**1. Answer any two questions** **2×2=4**

(a) State Gambler's ruin problem and write the transition matrix for it. **2**

(b) What do you mean by extinction probability? **2**

(c) Define the order of a Markov Chain. Discuss how a Markov Chain can be represented as a graph. **2**

(d) Define multiple correlation coefficient and partial correlation coefficient. **2**

**2. Answer any two questions** **4×2=8**

(a) State and prove the first entrance theorem. **4**

(b) State and prove Chapman Kolmogorov equation **4**

(c) Prove that the state *j* is persistent iff **4**

$$\sum_{0}^{\infty} p_{jj}^{(n)} = \infty.$$

(d) Consider the Markov Chain with transition probability matrix **4**

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Test whether the states are periodic and persistent.

**3. Answer any one question** **8×1=8**

(a) Find the differential equation for the Wiener process. **8**

(b) Considering appropriate assumptions derive the probability generating function for the birth and death process when birth and death rate respectively $n\lambda$ and $n\mu$, $n$ being the population size at time $t$ and $\lambda$ and $\mu$ are the constants . Assume that initial population size is $\boldsymbol{i}$. **8**

**[Internal Assesment-05 marks]**

# M.Sc. 4th Semester Examination, 2022
# Department of Mathematics,
# Mugberia Gangadhar Mahavidyalaya
# (Non-linear Optimization)
# Paper MTM – 404B

**FULL MARKS: 50 : : Time : 02 hours**

(*Candidates are required to give their answers in their own words as far as practicable*)

**1. Answer any four questions** **2×4=8**

(a) What do you mean by quadratic programming problem? **2**

(b) Write the primal and dual problems for unconstrained Geometric Programming problem. **2**

(c) What is non-vacuous matrix with example? **2**

(d) What is degree of difficulty in connection with Geometric programming. **2**

(e) Define Pareto optimal solution in a multi-objective non-linear programming problem. **2**

(f) What are the basic differences between the necessary criteria and sufficient criteria of Fritz-John saddle point theorem? **2**

**2. Answer any four questions** **4×4=16**

(a) Prove that all strategically equivalent bi-matrix game have the Nash equilibria. **4**

(b) Find equilibrium strategies in the game **Scissors-paper-stone:** **4**

| | | **Player 2** | | |
|---|---|---|---|---|
| | | Scissors | Paper | Stone |
| | Scissors | **(0, 0)** | **(1, -1)** | **(-1, 1)** |
| **Player 1** | Paper | **(-1, 1)** | **(0, 0)** | **(1, -1)** |
| | Stone | **(1, -1)** | **(-1, 1)** | **(0, 0)** |

(c) State and prove second existence theorem. **4**

(d) State Let $\theta$ be a numerical differentiable function on an open convex set $\Gamma\epsilon R^n$.Prove that necessary and sufficient condition that $\theta$ be convex on $\Gamma$ is that for each $x^1$, $x^2 \in \Gamma$, $[\nabla\theta(x^2) - \nabla\theta(x^1)]\ (x^2 - x^1) \geq 0$. **4**

(e) Derive the Kuhn-Tucker conditions for quadratic programming problem. **4**

(f) Write the relationship among the solutions of local minimization problem (LMP), the minimization problem (MP), the Fritz-John stationary problem (FJP), the Fritz-John saddle point problems (FJSP), the Kuhn-Tucker stationary point. **4**

**3. Answer any two questions** **8×2=16**

(a) Apply Wolfe's method for solving the QPP **8**

Max Z=$4x_1$+$6x_2$ -$2x_1^2$ -$2x_1x_2$ -$2x_2^2$

Subject to $x_1 + 2x_2 \leq 2$ and $x_1\ , x_2 \geq 0$.

(b) (i) State and prove Separation theorem of non-linear programming Problem. **4+4**

(ii) Solve the GPP problem, Minimize $f(x) = 7x_1{x_2}^{-1} + 7x_2{x_3}^{-2} + 5{x_1}^{-3}x_2x_3 + x_1x_2x_3$

(c) How do you solve the following geometric programming problem? Find $X = \begin{cases} x_1 \\ x_2 \\ \vdots \\ x_n \end{cases}$ that minimizes the objective function **8**

$f(x) = \sum_{j=1}^{n} U_j\,(x) = \sum_{j=1}^{N} (c_j \prod_{i=1}^{n} x_i^{a_{ij}})$ , $c_j > 0, x_i > 0, a_{ij}$ are real numbers, $\forall\ i, j$

(d) Using the chance-constrained programming technique to find an equivalent deterministic LPP of the following stochastic programming problem. **8**

$$Minimize\ F(x) = \sum_{j=1}^{n} c_j\,x_j$$

$$subject\ to\ P\left[\sum_{j=1}^{n} a_{ij}\,x_j \leq b_i\right] \geq p_i$$

$$x_j \geq 0, i, j = 1, 2, \cdots n$$

When $b_i$ is a random variable and $p_i$ is a specified probability.

**[Internal Assesment-10 marks]**

# M.Sc. 4th Semester Examination, 2022

## Department of Mathematics,

## Mugberia Gangadhar Mahavidyalaya

### (Operational Research Modelling-II)

## Paper MTM – 405B

**FULL MARKS: 25 : : Time : 01 hours**

(***Candidates are required to give their answers in their own words as far as practicable***)

**1. Answer any two questions** **2×2=4**

(a) Define reliability of a system. How does it differ from probability? **2**

(b) Define entropy function and explain its importance. **2**

(c) Discuss about the essential parts of communication system. **2**

(d) What is channel matrix? State fundamental theorem of information theory. **2**

**2. Answer any two questions** **4×2=8**

(a) A system is consisting of 4 identical subsystems connected in parallel. Each subsystem consists of 3 identical units connected in series. If the probability of reliable for each unit over a certain period of time is 0.95, obtain the system of reliability. **4**

(b) Show that R $(t)$ = exp $[-\int_0^t \lambda(t)dt]$, where $R(t)$ is the reliability function and $\lambda(t)$ represents the failure rate. What is mean time between failure? **4**

(c) For any two messages $X$, $Y$, prove that $H(X,Y) \leq H(X) + H(Y)$ with equality iff $X$ and $Y$ are independent. **4**

(d) State Pontryagin's Maximum Principle and explain it by an example. **4**

**3. Answer any one question** **8×1=8**

(a) A transmitter has a character consisting of five letters $\{x_1,\ \ x_2, \ldots\ldots, x_5\}$ and the receiver has a character consisting of four letters $\{y_1,\ \ y_2,\ y_3,\ y_4\}$. The joint probability for the communication is given below: **8**

| $p(x_i, y_j)$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|
| $x_1$ | 0.25 | 0 | 0 | 0 |
| $x_2$ | 0.10 | 0.30 | 0 | 0 |
| $x_3$ | 0 | 0.05 | 0.10 | 0 |
| $x_4$ | 0 | 0 | 0.05 | 0.10 |
| $x_5$ | 0 | 0 | 0.05 | 0 |

Determine the entropies H(X), H(Y) and H(X, Y).

(b) State and prove the additive property of entropy function. **8**

**[Internal Assesment-05 marks]**

**Problems on Lab for Operations Research of 4th Semester 2022**
**Subject MTM 495, B OR, Full Marks:-25**
**Group-A**

**Answer any one question** **1 x 6= 06**

**1.** Write a program and solve in LINGO to solve the following LPP using simplex method.

$$\text{Max } z = 3x_1 + 4x_2$$
$$\text{Subject to, } x_1 + x_2 \leq 10$$
$$2x_1 + 3x_2 \leq 18$$
$$x_1 \leq 8$$
$$x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

2. Write a program and solve in LINGO to solve the following QPP using Wolfe's modified simplex method.

$$\text{Max } z = 2x_1 + x_2 - x_1^2$$
$$\text{Subject to, } 2x_1 + 3x_2 \leq 6$$
$$2x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

**3.** Write a program and solve in LINGO to solve the following Geometric Programming Problem.

$$\text{Minimize } f(x) = 5x_1x_2^{-1}x_3^{2} + x_1^{-2}x_2^{-1} + 10x_2^{2} + 2x_1^{-1}x_2x_3^{-2}$$

**4.** Write a program and solve in LINGO to find the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

**5.** Write a program and solve in LINGO to solve the following Queuing theorem problem.

In a car wash service facility information gather indicates that cars arrive for service according to a Poisson distribution with mean 5 per hour. The time for washing and cleaning for each car varies but is found to follow an exponential distribution with mean 10 minutes per car. The facility can not handle more than one car at a time and has a total of 5 parking spaces. If the parking spot is full, newly arriving cars balk to 6 services elsewhere.

(a) How many customers the manager of the facility is loosing due to the limited parking spaces?

(b) What is the expected waiting time until a car is washed?

**6.** Write a program and solve in LINGO to solve the following problem of Inventory.

The demand for an item is deterministic and constant over time and is equal to 600 units per year. The unit cost of the item is Rs. 50.00 while the cost of placing an order is Rs. 100.00.The inventory carrying cost is 20% of the item and the shortage cost per month is Rs. 1.Find the optimal ordering quantity. If shortages are not allowed, what would be the loss of the company ?

**7.** Write a program and solve in LINGO to solve the following Stochastic Programming Problem.

A manufacturing firm produces two machines parts using lathes, milling machines and grinding machines. The machining times available per week on different machines and the machining times required on different machines for each part are given below. Assuming that the profit per unit of each of the machine parts I and II is a normally distributed random variable , find the number of machine parts to be manufactured per week to maximize the profit. The mean value and standard deviation of profit are Rs. 50 and 20 per unit for part I and Rs. 100 and 50 per unit for part II.

| Type of Machine | Machining time required per piece (minutes ) | | Maximum time available per week ( minutes ) |
|---|---|---|---|
| | Part I | Part II | |
| Lathes | $a_{11}=10$ | $a_{12}=5$ | $b_1=2500$ |

| | | | |
|---|---|---|---|
| Milling Machines | $a_{21}$=4 | $a_{22}$=10 | $b_2$=2000 |
| Grinding Machines | $a_{31}$=1 | $a_{32}$=1.5 | $b_3$=450 |

## Group-B

**Answer any one** **1X 9=09**

**8.** Write a program in MATLAB to solve the following QPP using Wolfe's modified simplex method.

$$\text{Max } z = 4x_1+6x_2-2x_1^2-2x_1x_2-2x_2^2$$
$$\text{Subject to, } x_1+2x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

**9.** Write a program in MATLAB to solve the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

**10.** Write a program in MATLAB to solve the following Queuing theorem problem.

Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes.

(a) What is the probability that a person arriving at the booth will have to wait?

(b) What is the average length of queues that form from time to time?

(c) The telephone company will install a second booth when convinced that an arrival would expect to have to wait at least 3 minute for the phone. By how much must the flow of arrivals be increased to justify a second booth?

(d) Find the average number of units in the system.

(e) What is the probability that an arrival has to wait more than 10 minutes before the phone is free?

(f) Estimate the fraction of a day that the phone will be in use (or busy).

**11.** Write a program in MATLAB to solve the following problem of Inventory.

An engineering factory consumes 5000 units of a component per year. The ordering, receiving and handling cost are Rs.300 per order while trucking cost is Rs.1200 per order, internet cost Rs. 0.06per unit per year, Deterioration and obsolence cost Rs 0.004 per year and storage cost Rs. 1000 per year for 5000 units. Calculate the economic order quantity and minimum average cost.

**12.** Write a program in MATLAB to solve the following Stochastic Programming Problem.

A manufacturing firm produces two machines parts using lathes, milling machines and grinding machines. The machining times available per week on different machines and the profit on machine part are given below. The machining times required on different machines for each part are not known precisely (as they vary from worker to worker) but are known to follow normal distribution with mean and standard deviations as indicated in the following table.

| Type of Machine | Machining time required per unit(minutes) | | | | Maximum time available per week (minutes) |
|---|---|---|---|---|---|
| | Part I | | Part II | | |
| | Mean | Standard deviation | Mean | Standard deviation | |
| Lathes | $\bar{a}_{11}=10$ | $\sigma_{a11}=6$ | $\bar{a}_{12}=4$ | $\sigma_{a12}=4$ | $b_1=2500$ |
| Milling machines | $\bar{a}_{21}=4$ | $\sigma_{a21}=6$ | $\bar{a}_{22}=10$ | $\sigma_{a22}=7$ | $b_2=2000$ |
| Grinding machine | $\bar{a}_{31}=1$ | $\sigma_{a31}=2$ | $\bar{a}_{32}=1.5$ | $\sigma_{a31}=3$ | $b_3=450$ |
| Profit per unit(Rs) | $c_1=50$ | | $c_2=100$ | | |

Determine the number of machine parts I and II to be manufactured per week to maximize the profit without exceeding the available machining times more than once in 100 weeks.

**13.** Write a program in MATLAB to solve the following LPP using simplex method.

$$\text{Max } z = 2x_1+3x_2-x_3$$
$$\text{Subject to, } 2x_1+5x_2-x_3 \leq 5$$
$$x_1+x_2+2x_3 = 6$$
$$2x_1-x_2+3x_3 = 7$$
$$x_1 , x_2 \geq 0$$

14. Write a program in MATLAB to solve the following QPP using Wolfe's modified simplex method.

$$\text{Max } z=2x_1+3x_2-x_1^2$$
$$\text{Subject to, } x_1+2x_2 \leq 4$$
$$x_1 , x_2 \geq 0$$

**15.** Write a program in MATLAB to solve the following Geometric Programming Problem.

$$\text{Minimize } f(x)=5x_1x_2^{-1} + 2x_1^{-1}x_2+5x_1+x_2^{-1}$$

**16.** Write a program in MATLAB to solve the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$\mathbf{A} = \begin{bmatrix} 8 & 0 \\ 30 & 2 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 8 & 30 \\ 0 & 2 \end{bmatrix}$$

**17.** Write a program in MATLAB to solve the following Queuing theorem problem.

A telephone exchange has two long distance operators. The telephone company finds that, during the peak load long distance all arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on this call is approximately exponentially distributed with mean length 5 minutes.

(a) What is the probability that a subscriber will have to wait for this long distance call during the peak hours of the day?

(b) If the subscriber waits and are serviced in turn, what is the expected waiting time.

**18.** Write a program in MATLAB to solve the following problems of Inventory.

A constructer has to supply 10,000 bearing per day to an automobile manufacturer. He find that when he start a production run, he can produce 25,000 bearing per day .The cost of holding a bearing in stock for one year is Rs 2 and set up cost for producing run is Rs 180. How frequently should the production ?

**19.** Write a program in MATLAB to solve the following Stochastic Programming Problem.

A manufacturing firm produces two machines parts using lathes, milling machines and grinding Write a program in MATLAB to solve machines. The machining times required on different machines for each part and the profit on machine part are given below. If the machining times available on different machines are probabilistic ( normally distributed ) with parameters as given in the following table , find the number of machine parts I and II to be manufactured per week to maximize the profit. The constraint have to be satisfied with a probability of at least 0.99.

| Type of Machine | Machining time required per piece (minutes ) | | Maximum time available per week ( minutes ) | |
|---|---|---|---|---|
| | Part I | Part II | Mean | Standard deviation |
| Lathes | $a_{11}=10$ | $a_{12}=5$ | $b_1=2500$ | $\sigma_{b1}=500$ |
| Milling Machines | $a_{21}=4$ | $a_{22}=10$ | $b_2=2000$ | $\sigma_{b2}=400$ |
| Grinding Machines | $a_{31}=1$ | $a_{32}=1.5$ | $b_3=450$ | $\sigma_{b3}=50$ |
| Profit per unit(Rs) | $c_1=50$ | | $c_2=100$ | |

**20.** Write a program in MATLAB to solve the following problem of Inventory.

The demand for an item in a company is 18000 units per year. The company can produce the item at a rate of 3000 per month. The cost of one set-up is Rs. 500 and the holding cost of one unit per month is Rs. 0.15.The shortage cost of one unit is Rs. 20 per month. Determine the optimum manufacturing quantity. Also determine the manufacturing time and the time between setup.

**Laboratory Note Book and Viva: 05**
**Field Tour with Report: 05**